Monte-carlo renormalization group study of gauged RP^2 spin models in two dimensions *

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The 2D RP^2 gauge model is studied using the Monte-Carlo Renormalization Group (MCRG). We confirm the first-order transition reported in [1] ending in a critical point associated with vorticity. We find evidence for a new renormalized trajectory (RT) which is responsible for a cross-over from the vortex dominated regime to the $O(3)$ regime as the coupling is reduced. Near to the cross-over region a good signal for scaling will be observed in RP^2 but this is illusory and is due to the proximity of the RT. We suggest that this is the origin of the 'pseudo'-scaling observed in [2]. We find that the continuum limit of RP^2 is controlled by the $O(3)$ fixed point.

1. Introduction

In [2] the mass gap measurement in the $SO(4)$ matrix model was compared with the Bethe-Ansatz prediction and found to disagree by a factor of four although for the covering group model there was good agreement. It was concluded that the signal for scaling in the $SO(4)$ simulation was only apparent and that a true continuum limit had not been achieved. It was conjectured that the deception was due to vortices, present in $SO(4)$ but absent in the covering group. Here we investigate whether vortices can cause a bogus signal for scaling in the 2D RP^2 gauge model which allows an interpolation between the pure RP^2 and the $O(3)$ spin models and which also contains vortices.

We find evidence for the $O(3)$ fixed point and for a scaling flow which indicates the presence of a new renormalized trajectory (RT). We suggest that this RT will cause an apparent scaling signal of the kind reported in [2] and that it is responsible for an observed cross-over effect.

It has also been conjectured [3] that in 2D the continuum limit in pure RP^2 is distinct from that in pure $O(3)$. Niedermayer et al. [4] and Hasenbusch [5] have suggested that this conjecture is incorrect and that the continuum limit in the RP^2 model is controlled by the $O(3)$ fixed point. The work we report here supports this conclusion.

The action used is

$$S \ = \ -\beta \left(\sum_{\boldsymbol{x},\boldsymbol{\mu}} \boldsymbol{S_x} \cdot \boldsymbol{S_{x+\mu}}\, \sigma_{\boldsymbol{x},\boldsymbol{\mu}} \ + \ \mu \sum_{P} P(\sigma) \right), \quad (1)$$

where $\boldsymbol{S_x}$ is a unit length three-component vector at site $\boldsymbol{x}$ and $\sigma_{\boldsymbol{x},\boldsymbol{\mu}}$ is a gauge field on the link $\boldsymbol{x},\boldsymbol{\mu}$ taking values in $[1,-1]$. The plaquette of gauge fields is denoted by $P(\sigma)$ and vortices reside on plaquettes where $P(\sigma)=-1$. The pure $O(3)$ and RP^2 spin models correspond to $\mu\to\infty$ and $\mu=0$ respectively.

A local update was used comprising a combination of heat-bath, microcanonical and demon schemes. Lattice sizes ranged from 64^2 to 512^2 with typically $\sim 5\cdot 10^5$ configurations per run. The simulations were carried out on the

*Presented by R.R. Horgan. This work is supported by NATO collaborative research grant no. CRG950234 and in part by the Leverhulme Trust.

HITACHI SR2201 computers in the Cambridge High Performance Computing Facility and in the Tokyo Computing Centre.

2. The MCRG scheme

To establish the topology of RG flows we study how the mean values of the spin-spin interaction, A, and of the plaquette, P, flow under blocking. For a given configuration these operators are given by

$$A = \frac{1}{2V}\sum_{\boldsymbol{x},\boldsymbol{\mu}} \boldsymbol{S_x}\cdot\boldsymbol{S_{x+\mu}}\,\sigma_{\boldsymbol{x},\boldsymbol{\mu}}\ ,$$
$$P = \frac{1}{V}\sum_{P} P(\sigma)\ . \qquad (2)$$

$\langle A\rangle \in [0,1]$, $\langle P\rangle \in [-1,1]$ and the vorticity is $\mathcal{V} = (1-\langle P\rangle)/2$.

For each configuration $\{\boldsymbol{S},\sigma\}$ on a lattice of side L we derive a blocked configuration $\{\boldsymbol{S}^B,\sigma^B\}$ on a lattice of side $L/2$. The blocking transformation for the spins is

$$\boldsymbol{S}^B_{\boldsymbol{x}_B} = \left(\boldsymbol{S_x} + \alpha\sum_{\nu}\boldsymbol{S_{x+\nu}}\sigma_{\boldsymbol{x},\boldsymbol{\nu}}\right)/\,|\ldots|\ .$$

where $\boldsymbol{\nu}$ runs over nearest neighbour displacements from $\boldsymbol{x}$, and α was chosen to be 0.0625 . To block the links the gauge field products were computed for the three shortest Wilson paths W_0, W_+, W_- joining the end points of the blocked link. The blocked gauge field was assigned the majority sign of the W_i .

For given coupling constants (β,μ) and given lattice size L^2 each configuration was blocked by successive transformations until the blocked lattice was 8^2. The operators $A_L(\beta,\mu)$ and $P_L(\beta,\mu)$ were then measured and averaged over all configurations. This was done for $L = 64, 128, 256, 512$ giving a flow segment in the $(\langle A\rangle,\langle P\rangle)$ plane with each point labelled by initial lattice size. The flow can be extended by tuning new couplings (β',μ') so that

$$A_{L'}(\beta',\mu') = A_L(\beta,\mu)\ ,$$
$$P_{L'}(\beta',\mu') = P_L(\beta,\mu)\ , \qquad (3)$$

for some L and L'. The segment for fixed (β',μ') can then be computed. These flows result from

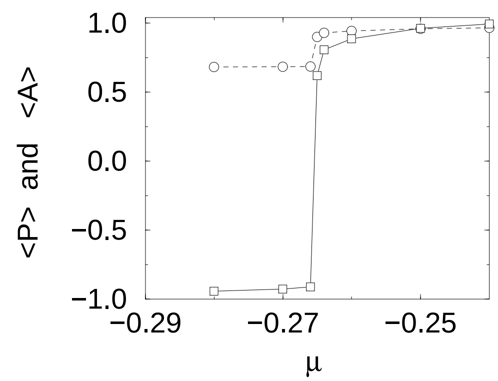

Figure 1. $\langle A\rangle$ (dashed) and $\langle P\rangle$ (solid) versus μ for $\beta = 8.0$ showing the first-order transition.

a projection onto the (β,μ) plane of true trajectories in a higher-dimensional coupling constant space. The assumption is that only A and P correspond to relevant operators and that by blocking the effect of irrelevant variables is minimized.

3. Results and Conclusions

There is a line of first-order transitions, first reported in [1], for $\mu \sim -0.3$ and $\beta \geq 7.5$. We find that this line of transitions ends at the "vorticity" critical point. In figure 1 we plot $\langle A\rangle$ and $\langle P\rangle$ versus μ for $\beta = 8.0$. The order parameter is the vorticity. The linear combination $U = 2A + \gamma P$ with $\gamma \sim -0.29$ shows *no* discontinuity in this region. U is closely related to the action, S (eqn. 1), and it is reasonable to interpret U as the free energy. Also, U is the correct operator to interpolate the vector state associated with the $O(3)$ critical surface where the corresponding correlation length will diverge but which remains *finite* at the vorticity critical point.

There is a set of neighbouring flows on which the observables scale. The example shown in figure 2 shows that there is a RT in the vicinity which must be associated with a new fixed point. To see that observables scale each flow segment of four points was successively overlaid using eqn. (3) with $L' = L/2$. For the scaling flows the points of the overlaid flow segments coincide very

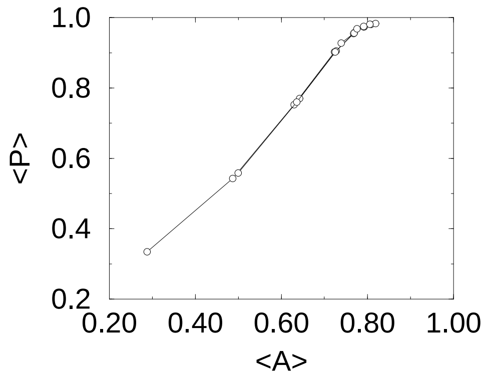

Figure 2. Scaling flow. All points are from the 8^2 blocking of, respectively, $L = 64, 128, 256, 512$.

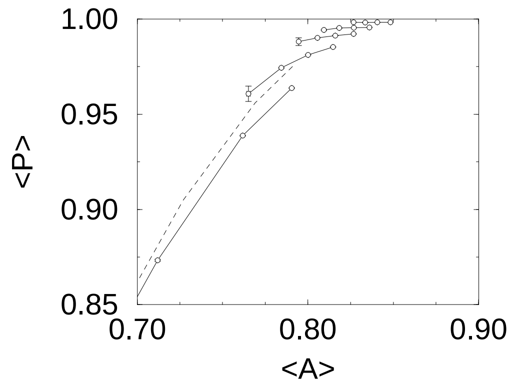

Figure 3. The cross-over region in RP^2 ($\mu = 0$) for $\beta = 3.9, 4.05, 4.17, 4.29, 4.5$.

well within errors. The flow is consistent with an exponent $\nu = 4.75(15)$ with critical couplings $(\beta^*, \mu^*) \approx (7.5, -0.3)$. Of course, the new fixed point will not lie in the (β, μ) plane and its relationship with the vorticity critical point is unclear. Open questions are: can the continuum limit of the new RT be taken at the vorticity critical point and does there exist a continuum limit with a non-zero vorticity density? It is intriguing that the position of the vorticity critical point and the critical couplings deduced from the fit for ν are very similar. The nature of the new fixed points needs more investigation.

Scaling on the $O(3)$ RT ($\mu = \infty$) was verified and agreement with the 3-loop perturbative $\beta-$function was found after effects due to finite L were accounted for using perturbation theory.

The pure RP^2 model ($\mu = 0$) does not intersect any critical surface except the one controlled by the $O(3)$ fixed point at $(\beta, \mu) = (\infty, \infty)$ ($(\langle A\rangle, \langle P\rangle) = (1, 1)$). This confirms the conjectures of [4,5] that RP^2 and $O(3)$ have the same continuum limit. In figure 3 flow segments for the RP^2 are shown for various β in the range 3.9 to 4.5. There is a clear cross-over in the blocked vorticity, $\mathcal{V}$, as β increases through this range. Moreover, the flows to larger $\mathcal{V}$ renormalize close to the scaling flow, shown in figure 3 as a dotted line. Hence, for $\beta \sim 4.0$ we should expect to see a good signal for scaling induced by the associated new RT. However, this scaling is not a signal for a continuum limit in RP^2 but is due to the proximity of the cross-over region to the new fixed point to which it is due. As β is increased through the cross-over region scaling will be violated and eventually reappear in association with the true continuum limit controlled by the $O(3)$ fixed point. We believe that this is the origin of the bogus scaling observed in [2].

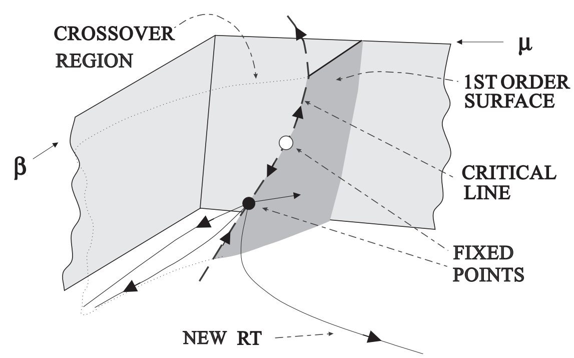

Figure 4. A line of critical points terminates the first-order surface and contains new fixed points from one of which flows the RT responsible for the observed cross-over behaviour.

In figure 4 we shown an artist's impression of a possible topology for the RG flows.

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